Prove that the following argument is valid using the Rules of Inference.
Give the reason for each step as shown in lecture.

SCORE: _____ / 9 PTS

[1] $\sim r \to p$
[2] $\sim s \vee w$
[3] $\sim p \wedge q$
[4] $(\sim q \vee r) \to s$
$\therefore w$

$\sim p \wedge q$	[3]
$\therefore \sim p$	SPEC
$\sim r \to p$	[1]
$\therefore r$	MT
$\therefore \sim q \vee r$	GEN
$(\sim q \vee r) \to s$	[4]
$\therefore s$	MP
$\sim s \vee w$	[2]
$\therefore w$	ELIM

EACH LINE WORTH 1

SUBTRACT $\frac{1}{2}$ FOR EACH MISSING REASON

(MUST BE IN THIS ORDER)

Let $A = \{-2, 1, 3\}$ and $B = \{-2, 0, 2\}$.

SCORE: _____ / 6 PTS

Let $P(x, y)$ be the predicate "$2x + y^2$ is a multiple of 3" with domain $A \times B$ (ie. $x \in A$ and $y \in B$).

Determine if the statement "$\forall x \in A, \exists y \in B : P(x, y)$" is true or false.

Justify your answer as shown in lecture. Use as few examples/counterexamples as possible.

FOR EACH $x \in A$

TEST $\exists y \in B : P(x, y)$

EITHER VALUE IS OK

$x = -2$ $\quad \exists y \in B : -4 + y^2$ IS A MULTIPLE OF 3 $\quad$ TRUE $y = 2$ OR -2

$x = 1$ $\quad 2 + y^2$ $\quad$ TRUE $y = 2$ OR -2

$x = 3$ $\quad 9 + y^2$ $\quad$ TRUE $y = 0$

TRUE

EACH ITEM WORTH $1\frac{1}{2}$

Let $P(x)$ be the predicate " x is a perfect square". Let $Q(x)$ be the predicate " $x^2 - 1$ is a multiple of 5 ". SCORE: _____ / 6 PTS

Let $D = \{1, 4, 5, 7, 9\}$ be the domain of both predicates.

[a] Find the truth set of $P(x)$. **You do NOT need to justify your answer.**

1 $\{1, 4, 9\}$

[b] Find the truth set of $Q(x)$. **You do NOT need to justify your answer.**

2 $\{1, 4, 9\}$

[c] Is the statement $P(x) \Rightarrow Q(x)$ true or false ? **Explain very briefly.**

$1\frac{1}{2}$ YES, TRUTH SET OF $P(x) \subseteq$ TRUTH SET OF $Q(x)$, $1\frac{1}{2}$

Consider the statement "De Anza students enrolled in at least 5 units are eligible for an Eco Pass". **SCORE: _____ / 6 PTS**

[a] Write the statement symbolically, using **TWO** predicates. **State clearly the domain and predicates.**

NO CREDIT IF PREDICATES DON'T HAVE "x"

$\frac{1}{2}$ $D = \{$DE ANZA STUDENTS$\}$

$\frac{1}{2}$ $P(x) =$ "x IS ENROLLED IN AT LEAST 5 UNITS"

$\frac{1}{2}$ $Q(x) =$ "x IS ELIGIBLE FOR AN ECO PASS"

1 $\forall x \in D,\ P(x) \rightarrow Q(x)$

[b] Write the negation of the statement symbolically, using the domain and predicates from [a].
Also, write that negation **informally (in natural sounding English)**.

$1\frac{1}{2}$

$\sim(\forall x \in D,\ P(x) \rightarrow Q(x)) \equiv \exists x \in D : \sim(P(x) \rightarrow Q(x)) \equiv \exists x \in D : P(x) \wedge \sim Q(x)$

THERE IS A DE ANZA STUDENT ENROLLED IN AT LEAST 5 UNITS WHO IS NOT ELIGIBLE FOR AN ECO PASS

1

[c] Write the inverse of the statement symbolically, using the domain and predicates from [a].

$\forall x \in D,\ \sim P(x) \rightarrow \sim Q(x)$ 1

Consider the statement "All calculus students have passed the same placement test." **SCORE: _____ / 5 PTS**

[a] Write the statement symbolically, using **TWO** variables. **State clearly the domains and predicate.**

NO CREDIT IF PREDICATES DON'T HAVE "S" AND "t"

$\frac{1}{2}$ $S = \{\text{CALCULUS STUDENTS}\}$, $P(s,t) =$ "s HAS PASSED t" 1

$\frac{1}{2}$ $T = \{\text{PLACEMENT TESTS}\}$, $\exists t \in T : \forall s \in S, P(s,t)$ $1\frac{1}{2}$

[b] Write the negation of the statement symbolically, using the domains and predicate from [a]. $1\frac{1}{2}$

$\sim(\exists t \in T : \forall s \in S, P(s,t)) \equiv \forall t \in T, \sim(\forall s \in S, P(s,t)) \equiv \forall t \in T, \exists s \in S : \sim P(s,t)$

Write the following statement **informally (in natural sounding English)**. SCORE: _____ / 3 PTS

Your answer should NOT use the phrases "for all", "for every", "for each", "for any", "such that", "there exists", "there is".

$\forall s \in S, \exists m \in M :\sim T(s, m)$ where S = set of all De Anza students, M = set of all De Anza math classes,

and $T(s, m)$ = "s has taken m"

NO DE ANZA STUDENT HAS TAKEN EVERY DE ANZA MATH CLASS.

GRADED BY ME